

CORRECTIONS TO “HOLOMORPHIC CURVES AND CELESTIAL MECHANICS”

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ABSTRACT. Here we make corrections to our discussion on Floer’s boundary operator, and take the opportunity to fix a misuse of terminology concerning exact symplectic diffeomorphisms.

1. CORRECTIONS

In Section 2 we outlined the construction of Floer’s chain complex in the symplectically aespical case. Consider a closed symplectic manifold (W, ω) such that ω and $c_1(TW, \omega)$ vanish on $\pi_2(W)$. Let H_t be a 1-periodic Hamiltonian on W and J_t be a 1-periodic ω -compatible almost complex structure. The vector field $-J_t(c(t))[\dot{c}(t) - X_{H_t}(c(t))]$ along a contractible closed loop c on W corresponds to the L^2 **gradient** of the action functional

$$\mathcal{A}_H(c) = \int_{\mathbb{D}} v^* \omega + \int_{\mathbb{R}/\mathbb{Z}} H_t(c(t)) dt$$

and not to the L^2 anti-gradient as we stated. Here $v : \mathbb{D} \rightarrow W$ is a capping disk for $c(t)$ and the L^2 -inner product is defined by

$$\langle X, Y \rangle_{L^2} = \int_{\mathbb{R}/\mathbb{Z}} \omega(c(t))(X(t), J_t(c(t))Y(t)) dt$$

for all vector fields X, Y along the loop c . Moreover, for generic data (H, J) , the boundary operator is

$$\delta_{H,J}(x) = \sum_{\text{CZ}(y)=\text{CZ}(x)-1} (n(x, y) \bmod 2) y$$

where $n(x, y)$ is a count of solutions $u(s, t)$ of Floer’s equation satisfying

$$\lim_{s \rightarrow +\infty} u(s, t) = x(t) \quad \lim_{s \rightarrow -\infty} u(s, t) = y(t)$$

In the published version of the paper we interchanged the positions of the asymptotic limits. This completes our set of corrections to the discussion on Floer’s chain complex.

Finally, in Section 1 there is a misuse of the expression “center of mass”. A diffeomorphism Ψ of $\mathbb{R}^{2n}/\mathbb{Z}^{2n}$ isotopic to the identity can be lifted to a diffeomorphism $\tilde{\Psi}$ of the universal covering \mathbb{R}^{2n} and the integral in equation (8)

$$\int_{[0,1]^{2n}} \Delta(z)$$

is not the center of mass, it is the **shift of the center of mass** by the lifted map $\tilde{\Psi}$. There are a few places where this erroneous use of terminology needs correction. For instance, in the definition of exact symplectic diffeomorphisms condition (H3) should read “ Ψ is exact: it admits a lift to \mathbb{R}^{2n} that preserves center of mass”. This is in accordance to the discussion in [1, appendix 9], where the expression “center of gravity” is used.

REFERENCES

- [1] V.I. Arnold, *Mathematical methods of classical mechanics*. Springer-Verlag, Berlin and New York, 1978.